Some observations regarding the Matts-Öhnfeldt equation

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Abstract — This paper presents three observations regarding the well-known Matts-Öhnfeldt formula for collection efficiency of electrostatic precipitators. The Matts-Öhnfeldt equation is universally used for ESP sizing and evaluation of ESP performance and relates the ESP collection efficiency to the (modified) migration velocity, w_k, and the specific collecting area, A/Q, via $\eta = 1 - \exp[-(w_k A/Q)^k]$. First it is pointed out that the Matts-Öhnfeldt equation is identical to the famous Weibull distribution function, which is used frequently in reliability theory and many other scientific areas. Second, it is shown that there exists a particle size distribution where, under the normal approximation of proportionality between particle diameter and migration velocity, the resulting ESP efficiency is exactly the Matts-Öhnfeldt formula. Finally, cases where the parameter k in the Matts-Öhnfeldt equation is larger than 1 are discussed and exemplified.

Keywords — Electrostatic precipitator, ESP, migration velocity, Matts-Öhnfeldt

I. INTRODUCTION

The base for all experimental and theoretical evaluation of electrostatic precipitator (ESP) performance is the Deutsch equation [1-3]:

$$\eta = 1 - \frac{C_{out}}{C_{in}} = 1 - \exp\left[-wA/Q\right]. \qquad (1)$$

At ideal conditions Deutsch' equation relates the collection efficiency, η, of the ESP to the particle migration velocity, w, and the specific collecting area A/Q (square meters of ESP collecting area divided by the treated gas volume in cubic meters per second). One of the main assumptions is that all particles are identical, which leads to severe limitations of the usefulness of the equation for most practical cases. For mono-disperse (uniform) particle size distributions, or at least a very narrow width of the distribution, Deutsch' equation is applicable, while it quickly loose its validity as the particle size distribution widens. This was explicitly demonstrated in a paper by Allander and Matts in 1957 [4].

Since Deutsch' equation is still a reasonable approximation for the individual size fractions the total ESP collection efficiency for any size distribution, $\gamma(D)$, can in principle be obtained by integration over the entire particle size range [3-4]:

$$\eta = 1 - \int_0^\infty \gamma(D) \exp\left[-w(D)A/Q\right] dD . \qquad (2)$$

This, however, requires detailed knowledge of the exact particle size distribution, which is rarely the case. Furthermore the functional dependence of migration velocity as function of particle diameter, i.e. $w(D)$, must be inserted. Should the size distribution be known integration of Eq. (2) is readily performed by numerical methods, but it typically denies an explicit analytic expression for the relation between collection efficiency, specific collecting area and migration velocity.

In 1964 Matts and Öhnfeldt presented a modified Deutsch formula for precipitator sizing, which despite its simplicity and only two fitting parameters turned out to be very useful for describing the ESP efficiency as function of specific collecting area [5]. The Matts-Öhnfeldt equation introduces the modified migration velocity, w_k, and a shape- or dampening parameter k,

$$\eta = 1 - \exp\left[-\left(w_k A/Q\right)^k\right]. \qquad (3)$$

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Via the k-parameter, which is typically below one, the equation simulates the increased difficulty to collect remaining dust as the precipitator becomes larger to reach very high collection efficiencies. This phenomenon is typically due to decreasing average particle size, or more general that the most difficult-to-catch particles survives the longest in the ESP and are enriched. For fly ash precipitation after coal-fired boilers the value of $k = 0.5$ has turned out to a reasonably good approximation for a wide variety of fuel and process conditions. For the special case of a very narrow, or even mono-disperse, particle size distribution the value of k can under ideal conditions approach one, and the Matts-Öhnfeldt formula collapses to the ordinary Deutsch equation.

In this paper some observations on the Matts-Öhnfeldt formula of general interest will be pointed out and described. After looking at the differential equations that result in the Deutsch and Matts-Öhnfeldt formulas (Sec. II), three different properties of the Matts-Öhnfeldt equation are discussed. In Sec. III it is noted that the Matts-Öhnfeldt formula is a Weibull distribution with concentration of particulate matter as a function of treatment time. This renders some further insight and credibility to the Matts-Öhnfeldt formula, on account of the very wide use of Weibull distributions in many scientific areas. In Sec. IV the questions is asked whether there exist a particle size distribution that will result in the Matts-Öhnfeldt equation if integrated over all particle sizes using Deutsch equation for the fractional collection efficiency. Relatively simple mathematical considerations show that this is indeed the case, at least for the common case of $k = 0.5$, and that the resulting explicit expression for the size distribution reasonably well resembles a log-normal distribution in certain ranges. Finally, in Sec. V, the theoretical basis for k-values larger than one in the Matts-Öhnfeldt formula is discussed. An actual example with $k > 1$ is provided in the form of precipitation of oil soot after a small oil-fired boiler.

II. DIFFERENTIAL EQUATIONS FOR THE DEUTSCH AND MATTS-ÖHNFELDT FORMULAS OF ESP EFFICIENCY

Detailed derivations of the ordinary Deutsch equation are of course provided in the literature, for example in Harry White's book [3]. A simple derivation of Deutsch equation can for example proceed as follows.

Consider a thin slice of the dust laden gas inside the ESP, covering the entire cross-section and moving from the inlet towards the outlet at velocity v. No gas or particles are exchanged between adjacent slices, but it is assumed that full turbulent mixing takes place within each slice, leading to random motion of the particles perpendicular to the main gas flow direction and rapid loss of memory of their previous position. It is furthermore assumed that there is no inherent time dependence in the precipitation process and no explicit or implicit variation along the length of the ESP. A particle in the slice then has constant probability per time unit to be collected, and the decrease in particle concentration due to precipitation becomes proportional to the concentration itself:

$$\frac{d\,\mathrm{C}(t)}{dt} = -f\,\mathrm{C}(t)\,. \qquad (4)$$

Here f is a constant and $\mathrm{C}(t)$ is the concentration of particles suspended in the flue gas. The constant f has dimension s^{-1}, and may be viewed as a "precipitation probability per time unit", such that $f dt$ is the fraction of particles separated from the flue gas in the time interval dt. Solving Eq. (4) with the boundary condition at the ESP inlet $t = 0$ and $\mathrm{C}(0) = C_{in}$, we obtain the solution:

$$\mathrm{C}(t) = C_{in}\exp\left[-ft\right]. \qquad (5)$$

Although expressed in terms of treatment time in the ESP, Eq. (5) is identical to Deutsch' equation, Eq. (1). To go from treatment time to position along the length of the ESP we can replace t with x/v. Furthermore, the gas velocity through the ESP, v, is given by Q/Ω, where Q is the volumetric flow rate and Ω is the ESP cross-section. At the ESP outlet, at position $x = L$, the total treatment time is $T = L\Omega/Q = V/Q$, where V is the volume of the ESP. Inserting into Eq. (5) we get:

$$\mathrm{C}(L) = C_{out} = C_{in}\exp\left[-fV/Q\right]. \qquad (6)$$

We remember that $f\,dt$ is the fraction of particles collected in any of the "gas slices" during the time interval dt. This fraction is the portion of particles that are sufficiently close to the collecting electrode surface to be precipitated. With a particle migration velocity, w, towards the collecting surface the fraction of particles that are precipitated is $Awdt/V$ (the thickness of the slice in relation to the entire length of the ESP cancels). Identifying $f = Aw/V$ and substituting into Eq. (6) we obtain the usual Deutsch equation in terms of specific collecting area, Eq. (1).

One of the main assumptions in the analysis above, arriving at Deutsch' equation, is that the probability of precipitation, f, (or migration velocity w) is constant in time and space. Due to a non-uniform particle size distribution and size dependent migration velocity this is not the actual case. We can lift this restriction with an *ad hoc* approach by changing the constant f to a function with explicit time dependence $f \rightarrow \mathrm{f}(t)$. We then obtain the differential equation:

$$\frac{d\,\mathrm{C}(t)}{dt} = -\mathrm{f}(t)\,\mathrm{C}(t)\,. \qquad (7)$$

To get a simple differential equation we take $\mathrm{f}(t)$ to be of the form bt^{α}, which can cover both increasing and decreasing collection rates by the choice of the parameter α. For the special case $\alpha = 0$ and with the constant $b = f$, we arrive again at a constant collection probability and the differential equation leading to the Deutsch formula. The separable differential equation

$$\frac{d\,\mathrm{C}(t)}{dt} = -bt^{\alpha}\,\mathrm{C}(t)\;, \qquad (8)$$

is readily integrated. For $-1 < \alpha < \infty$ and $b > 0$, and denoting again the concentration at $t = 0$ (i.e. at the ESP inlet) by C_{in}, we obtain the solution:

$$\mathrm{C}(t) = C_{in}\exp\left[-\frac{b}{\alpha+1}t^{\alpha+1}\right]. \qquad (9)$$

We may rewrite Eq. (9) by the substitutions $k = \alpha + 1$ and $f_k = (b/(\alpha+1))^{1/(\alpha+1)}$ to get the more familiar expression:

$$\mathrm{C}(t) = C_{in}\exp\left[-\left(f_k t\right)^k\right]. \qquad (10)$$

This is of course the Matts-Öhnfeldt equation expressed in terms of treatment time. As before, we may replace time with position by dividing with the net gas flow velocity and evaluating the equation at the ESP outlet $x = L$. Analog with Eq. (6) the outlet particle concentration is then expressed in terms of the ESP volume:

$$\mathrm{C}(L) = C_{out} = C_{in}\exp\left[-\left(f_k V/Q\right)^k\right]. \quad (11)$$

By *defining* a new constant, $w_k = f_k V/A$, we retrieve the conventional form of the Matts-Öhnfeldt equation. Since w_k has the dimension m/s, we may call it "modified migration velocity", but it no longer represents the actual speed of the particles towards the collecting surface, as when deriving Deutsch' equation.

A point of notice during the "derivation" of Matts' and Öhnfeldt's modified Deutsch equation above concerns the value of the parameter α. When selecting a value of α in the range $-1 < \alpha < 0$, the value of $\mathrm{f}(t)$ in Eq. (7) is infinite at $t = 0$. While this may seem somewhat awkward it is mathematically of no consequence since $\mathrm{f}(t) = bt^{\alpha}$ has a finite integral in the range 0 to T when α is larger than -1. Also from physical point of view it is not completely unrealistic since there is an immediate sizable fallout of coarse dust in the first few inches of the ESP, even before reaching the corona region. Another point that deserves some elaboration is the selection of a function, $\mathrm{f}(t)$, with explicit time-dependence in the model. It should be clear that this is not formally correct, since the real dependence on treatment time or position is due to an underlying change in concentration, such that $f \rightarrow \mathrm{f}(\mathrm{C}(t))$ would be more appropriate. Nevertheless, the use of the form $\mathrm{f}(t)$ has it merits, and is conceptually simple and has some pedagogical value. The approach where $\mathrm{f}(\mathrm{C}(t))$ is used in lieu of $\mathrm{f}(t)$ in Eq. (7) is discussed in an Appendix.

III. MATTS-ÖHNFELDT EQUATION AS WEIBULL DISTRIBUTION

In 1951 the mathematician Waloddi Weibull presented and described a statistical distribution that subsequently became known as the Weibull distribution [6]. First he noted that the cumulative distribution function for any parameter of interest may be written in the form $F(x) = 1 - \exp[-\varphi(x)]$, where $F(x)$ is the fraction of the population having a parameter value lower than x. This is a suitable form of writing since the multiplication of probabilities of random or very complex events in a process are transformed to summations in the exponent. The only general conditions that $\varphi(x)$ has to satisfy is to be zero up to some value x_u and thereafter strictly non-decreasing. By taking the simplest class of functions that satisfies this criteria, $(x-x_u)^m/x_0$, Weibull presented the distribution function

$$F(x) = 1 - \exp\left[-\frac{(x-x_u)^m}{x_0}\right]. \qquad (12)$$

In his 1951 paper Weibull immediately went on to show the usefulness of the proposed statistical distribution by fitting the measured data for a number of examples, including fiber strength of cotton, fatigue life of steel and particle size of fly ash. Nowadays the Weibull distribution is one of the most widely used distribution functions, described in most mathematical handbooks and included in numerous software tools. Several recent books deals with the Weibull distribution in great detail [7-9]. The classical application of the Weibull distribution is in survival and reliability analysis, but it has also turned out to be useful for such disparate applications as dielectric breakdown voltages, pitting corrosion, thermo-luminescence glow peaks, wind speed distributions, rainfall intensity, cancer clinical trial data, stock returns, sampling plans in quality control, inventory lead times, warranty periods, tree diameter data and earthquake occurrences [9].

In addition to all the applications listed in Refs. 7-9, the Weibull distribution is obviously also suitable to describe the collection efficiency of dust in an ESP, since it is in fact identical to the Matts-Öhnfeldt equation. More precisely, it replicates the Matts-Öhnfeldt equation with $x_u = 0$, $m = k$ and $x_0 = w_k^{-k}$. The special case of $x_u = 0$ is normally referred to as the two-parameter Weibull distribution and is a very common case, especially for time-dependent processes where it is possible to identify a well-defined starting time. The two-parameter Weibull distribution is plotted in Fig. 1 for some values of the shape parameter m.

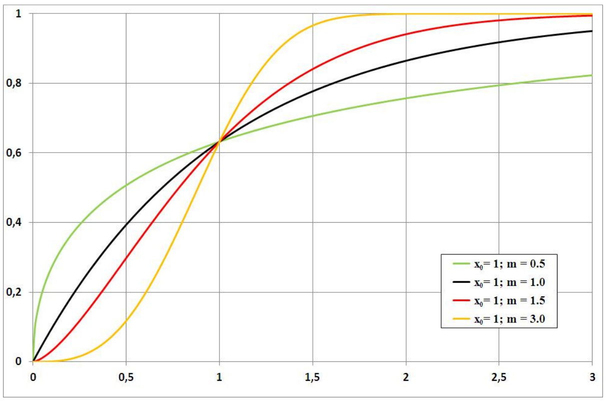

Fig. 1. Two-parameter Weibull distribution functions for different values of the shape parameter m and normalized scale parameter x_0.

The observation that the Matts-Öhnfeldt equation is nothing but a Weibull distribution applied to the mass concentration of dust inside an ESP is by no means of fundamental character. The Weibull distribution as well as the Matts-Öhnfeldt equation were already in their original papers (Ref. 5 and Ref. 6 respectively) clearly stated to be nothing more than practical tools to fit and extrapolate experimental data. As such, however, they turned out to be extremely useful over the years. Due to the wide applicability of the Weibull distribution in so very many technical and scientific applications it does provide some justification for the Matts-Öhnfeldt formula as a high-level representation of the underlying complex precipitation process. Since the Weibull distribution is extremely well-investigated and described in the literature, it is likely that some of the methods developed for its analysis may also turn out useful in the study of electrostatic precipitation.

A couple of previous applications of the Weibull distribution deserve special comments since they are in certain aspects somewhat similar to the electrostatic precipitation mechanism. One scientific discipline where the Weibull distribution is extensively used to model an inherently complex phenomenon is the *in vitro* dissolution of drugs [10-11]. Exactly as for electrostatic precipitation it is the matter of a mass transfer process, where a changing particle size over time is one of the dominating factors for the rate of transfer. Also the mass transfer of water in food drying and rehydration processes has turned out to be well described by the

Weibull distribution [12-14]. Finally, rate equations for the yield of heterogeneous chemical reactions have also been expressed as Weibull distributions [15].

IV. RELATION BETWEEN PARTICLE SIZE DISTRIBUTION AND MODIFIED MIGRATION VELOCITY

Typically the dust emanating from various processes, such as e.g. combustion in a coal-fired boiler, is assumed to have a particle size distribution that is approximately log-normal [3]. In reality combustion processes often result in bimodal or multimodal size distributions, reflecting the various generation mechanisms of particulate matter in the flame zone. The log-normal distribution may still be used for the individual modes of the distribution, and certainly for the coarsest mode that contains a very large percentage of the total particle mass in the flue gas. This coarsest mode of particular matter, where field charging is the dominating charging mechanism in the ESP, was the focus of Matts and Öhnfeldt when they developed the modified Deutsch equation. In the field charging regime the electrical charge, and hence the driving force, for a particle is proportional to its surface area. The theoretical formula for the charge together with the formula for the Stoke's drag force result in the familiar expression for the migration velocity in the field charging regime [3-4]:

$$w(D) = \frac{\varepsilon}{\varepsilon + 2} \frac{\varepsilon_0 E_0 E_p D}{\mu} . \qquad (13)$$

With the proportionality between migration velocity and particle diameter the integration over all sizes to obtain the collection efficiency according to Eq. (2) becomes more tractable. Combining Eq. (2) and Eq. (13) we obtain

$$\eta = 1 - \int_0^\infty \gamma(D) \exp[-sD] dD , \quad (14)$$

where the introduced factor s includes A/Q and the coefficients of Eq. (13). For the case when $\gamma(D)$ is the log-normal distribution the integral in Eq. (14) has no analytical solution but can be computed numerically, which was done in Refs. 4-5. By plotting such numerical solutions Matts and Öhnfeldt could demonstrate that their modified Deutsch equation gave a qualitatively similar behaviour [5]. Good agreement with experimental data from coal-fired plants was obtained with a value of the parameter k equal to 0.5.

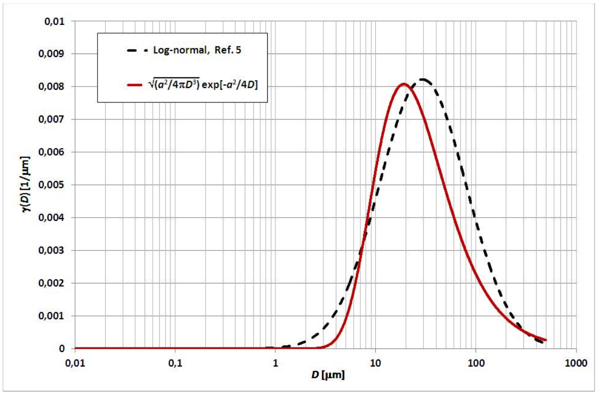

Fig. 2. Particle size distribution that results in the Matts-Öhnfeldt formula (a^2 = 115 μm), compared to the log-normal distribution from Ref. 5.

Here we shall pose the opposite question of the observed good agreement between the Matts-Öhnfeldt equation and integration of the log-normal distribution in the field charging regime: Does there exist a particle size distribution (possibly similar to the log-normal distribution) that gives the Matts-Öhnfeldt formula when integrated according to Eq. (14)? An observation that turns out helpful to answer this question, and which may also be of more general use, is that the integral in Eq. (14) is the definition of the Laplace transform of γ(D). Thus the rules for Laplace transforms and tabulated transform pairs may be generally used to analyse the total ESP efficiency as long as the charging mechanism is dominated by field charging (i.e. the migration velocity is proportional to particle diameter). If we limit our self to the special case of the Matts-Öhnfeldt equation where k = 0.5, which is one of the most used values of the k-parameter, we find in ordinary tables of Laplace transforms:

$$\int_0^\infty \frac{\exp[-a^2/4t]}{\sqrt{\frac{4\pi}{a^2} t^3}} \exp[-st] dt = \exp\left[-\sqrt{a^2 s}\right] . \quad (15)$$

In other words the particle size distribution $\gamma(D) = \sqrt{(a^2/4\pi D^3)}\exp[-a^2/4D]$ results in a total collection efficiency identical to the Matts-Öhnfeldt formula, with k = 0.5 and $w_k = a^2\varepsilon(\varepsilon+2)^{-1}\varepsilon_0 E_0 E_p/\mu$. It is now of interest to plot this particle size distribution for various values of its only parameter, a. The result may be done as an exercise and reveals that the shape of the size distribution function is quite reasonable. In Fig. 2 it is plotted for the case a^2 = 115 µm, for which it is reasonably similar to the particular log-normal distribution that Matts and Öhnfeldt used as example in Ref. 5 (also shown in Fig. 2).

V. ELECTROSTATIC PRECIPITATION WITH k-VALUE LARGER THAN ONE

Often in the literature it is stated or implicitly understood that the k-parameter in the Matts-Öhnfeldt modified Deutsch equation is below one or, for a virtually mono-disperse size distribution, equal to one [16-17]. While it was shown above that such a statement has theoretical justification under ideal conditions and with a particle size distribution where field charging is dominating, it will here be discussed why there are cases with $k > 1$. The physical meaning of a k-value larger than one is that the precipitation rate or efficiency increases with the length of the precipitator. At a first glance this is counter-intuitive, considering the fundamental principle that the easiest-to-collect particles are precipitated first, enriching the most difficult particles towards the exit of the ESP. Nevertheless there are theoretical justifications as well as field measurements that support increasing collecting rate with increasing treatment time, i.e. efficiencies governed by the Matts-Öhnfeldt formula with a k-parameter larger than one.

Consider a particle size distribution that is very narrow and located to a large extent in the sub-micron range. This could for example be the very fine soot from combustion of oil or Orimulsion in a boiler or large diesel engine. For such a size distribution the difference in migration velocity between different particles is quite small since it is narrow and located close to the minima of the fractional collection efficiency. Thus, when the flue gas is progressively cleaned as it passes through the ESP, there is very limited enrichment of particles that are more difficult than average to precipitate. This would in an ideal situation correspond to a k-value very close to 1. In reality, however, there are some factors that explicitly depend on the position or treatment time in the ESP, such as uneven gas distribution and corona suppression. These become less pronounced as the gas moves through the ESP.

It is well known that the gas flow profile of the ESP cross-section becomes more uniform as the flue gas moves through the ESP. A more even gas flow improves the overall collection efficiency (which can be interpreted as a higher migration velocity) and also reduces non-ideal effects such as sneakage and re-entrainment of dust. Another important practical problem, especially for dust with a high fraction of fines, is the corona suppression, or space charge effect [18-19]. Due to the very large surface area of the suspended dust particles their saturation charge is high enough to quench the corona current to levels much lower than in a dust free environment. Both the lower current and the fact that the particles cannot reach their saturation charge before a large fraction of the dust has been collected, leads to lower migration velocity in the front of the ESP compared to the rear sections. Even aside from the space charge effect it must be considered that sub-micron particles are dominated by the diffusion charging mechanism, which requires longer time to reach saturation charge. Thus, as opposed to the field charging where the particles can reach saturation charge in less than a second, sub-micron particles increase their electrical charge basically all the way through the ESP [3].

The above discussion imply that there are several possible cases where a very limited enrichment of difficult-to-catch particles can be dominated by factors that benefit from increased ESP length and increased treatment time. This translates to a migration velocity that effectively increases with increasing treatment time, i.e. a modified migration velocity according to the Matts-Öhnfeldt equation where the k-parameter is larger than one. The cases where this can be expected to occur are processes generating dust in a narrow size range with a lot of sub-micron particles. The precipitated dust should also have fairly low resistivity, so that no increase in back-corona propensity takes place in the downstream sections of the ESP. One actual example is provided below in the form of a pilot ESP treating flue gas from a small oil-fired boiler.

In 2005 Alstom installed an ESP based on the so-called ERDEC design downstream a

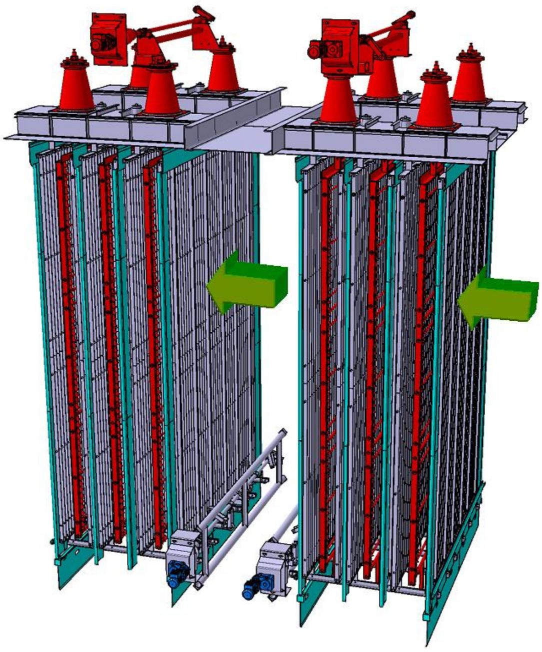

Fig. 3. Drawing of the internals of the cross flow Erdec-design ESP at KKAB power plant. The direction of the gas flow is indicated by the green arrows.

12 MW_{th} auxiliary oil-fired boiler at the Karlshamn Kraft (KKAB) power plant in Sweden [20]. The layout of the electrodes in the ERDEC design is different compared to a normal ESP. Being of a cross-flow design, the flue gas passes several modules consisting of discharge electrodes sandwiched between an upstream and downstream collecting electrode system. Both the saw tooth shaped discharge electrodes and the flat bars that constitute the collecting system are oriented in the same direction as the gas flow. This means that the electric field lines and the corona current are parallel to the gas flow direction, as opposed to the conventional duct-type ESP where they are basically perpendicular to the gas flow. The electrode layout inside the ERDEC design ESP at KKAB is shown in Fig. 3. Each of the two fields consists of three modules having one front and one rear collecting system with the discharge electrodes in between.

While the ERDEC design ESP at KKAB was a commercial project with standard contracts and requirements, it was also a demo installation where some pilot tests were performed in cooperation with the customer. The results from one of the test measurements performed at KKAB are presented in Table I. For this particular test the boiler was firing heavy fuel oil with a sulphur content of 2.1%. The centrifugal burner in the boiler had just

TABLE I
Gravimetric measurement results at the inlet and outlet of the ERDEC design ESP at KKAB power plant.

Test	ESP current	Inlet dust	Outlet dust
1	27 mA / 79 mA	15.0 mg/Nm^3	1.6 mg/Nm^3
2	29 mA / 70 mA	15.6 mg/Nm^3	1.3 mg/Nm^3
3	30 mA / 68 mA	15.5 mg/Nm^3	2.3 mg/Nm^3
4	0 mA / 37 mA	14.7 mg/Nm^3	7.0 mg/Nm^3

been cleaned and overhauled, resulting in very low particulate concentration in the flue gas. During the entire test day the inlet dust load to the ESP was very close to 15 mg/Nm^3 with only minor fluctuations. Despite the low concentration the ESP experienced significant corona suppression, as can be appreciated from the difference in current input between the first and second field, shown in Table I (tests 1-3). This is a certain sign that the particles generated in the boiler are mainly sub-micron, or otherwise the low concentration would never result in noticeable corona suppression. With both field 1 and field 2 in operation the average dust emission at the ESP outlet was 1.7 mg/Nm^3. In the last test the first field was put out of service, resulting in an increase in emission to 7.0 mg/Nm^3 (only one isolated measurement). Since the gas flow was kept constant during the entire measurement day, switching off one field corresponds to reducing the specific collecting area (and treatment time) to half.

With two operating conditions at different specific collecting area it is possible to fit the two parameters w_k and k of the Matts-Öhnfeldt equation. Due to the unconventional electrode configuration of the ERDEC design, the definition of total collecting area is not as straightforward as in a conventional ESP. It may be defined in several ways, but in the following analysis we shall define it simply as the sum of the actual area of all the flat bars in the collecting frames. Another definition could be twice the ESP cross section times the number of discharge electrode frames, or the performance equation could be based on treatment time in the electric field. In the end the definition of collecting area is rather unimportant as it only represent a scaling and redefinition of the modified migration velocity, while leaving the k-parameter unchanged. With the collecting area defined as the total flat bar area, operation with

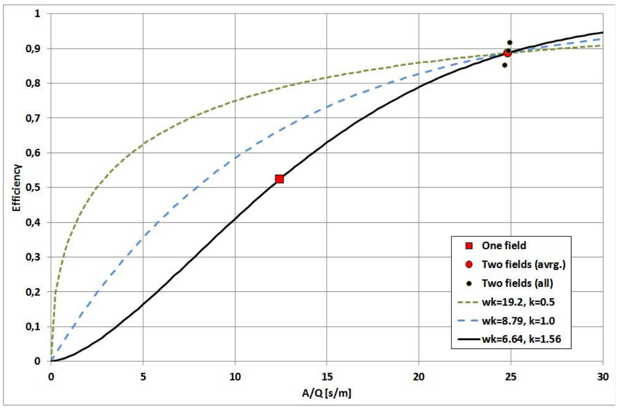

Fig. 4. Measured ESP efficiencies at KKAB with one and two fields in operation, together with curves from the Matts-Öhnfeldt equation for different values of the parameter k.

one and two fields represented a specific collecting area of 12.4 and 24.8 $m^2/m^3/s$, respectively. Fitting the measured collection efficiencies to the Matts-Öhnfeldt formula results in a modified migration velocity, w_k, of 6.64 cm/s, with the parameter k being 1.56.

The collection efficiency according to the Matts-Öhnfeldt equation with $k = 1.56$ and $w_k = 6.64$ cm/s is plotted in Fig. 4 together with the measured results. For comparison also curves for the Matts-Öhnfeldt equation with $k = 1$ and $k = 0.5$ are included. The corresponding values of w_k (8.56 and 19.2 cm/s respectively) have been selected to match the measured efficiency with two fields in operation. It is clearly seen that only the case with $k > 1$ is able to correctly extrapolate the trend to the situation when A/Q is decreased to half due to one field out of service.

VI. CONCLUSION

The Matts-Öhnfeldt equation has proven to be a useful practical tool for ESP sizing and performance evaluation for more than 50 years. Despite only two parameters it can accurately fit and extrapolate experimental data of ESP collection efficiency from a wide variety of process condition and dust types. The same type of equation, universally referred to as the Weibull distribution, has also turned out to be able to fit data from a multitude of scientific problems. In the present paper some further evidence on the soundness and flexibility of the Matts-Öhnfeldt equation has been provided. Despite ever increasing capabilities of computer simulations it is believed that field experience from full-scale ESP installations and data from pilot investigations is still the most important source for predictions of the very complex process of electrostatic precipitation. As such, the Matts-Öhnfeldt equation still has its place as a simple and transparent expression for evaluation and sizing of ESP installations.

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APPENDIX

As mentioned in Sec. II, a conceptually more appealing model to simulate the increasing difficulty to collect finer particles results if one considers the differential equation

$$\frac{d\,\mathrm{C}(t)}{dt} = -\mathrm{f}\big(\mathrm{C}(t)\big)\mathrm{C}(t)\,. \qquad \text{(A1)}$$

An equation like this means that the rate of precipitation depends on the remaining concentration of particles. The mechanism is that when the concentration reduces as time goes the character of the remaining particles has changed, affecting the precipitation rate. As in Sec. II we now limit the study to some simple functional relationship, which admits analytic solution. It may be assumed that the "coefficient" f(C(t)) will not vary within an extremely wide range, while C(t) itself will have some sort of exponential decay and vary within several orders of magnitude. Thus it is likely convenient to express the functional relationship f(C) on a logarithmic scale, and we may write:

$$\frac{d\,\mathrm{C}(t)}{dt} = -\mathrm{f}\big(\ln(\mathrm{C}(t))\big)\mathrm{C}(t)\,. \qquad \text{(A2)}$$

Since the function f is anyhow not defined, the rewrite from (A1) to (A2) is so far done without loss of generality. Now, however, we will limit the analysis to a special form of functional relationship, namely:

$$\mathrm{f}\big(\mathrm{C}(t)\big) = a\big(-\ln(\mathrm{C}(t)/C_{in})\big)^{\beta}\,. \qquad \text{(A3)}$$

The selection of functional form is basically analog to what was done in Sec. II, and we have normalized the concentration to the inlet dust load, C_{in}. The minus sign makes the argument positive and strictly increasing when C(t) decreases.

With the form f(C) according to Eq. (A3) the separable differential equation (A1) becomes:

$$-\int_{C_{in}}^{\mathrm{C}} \frac{d\,\mathrm{C}}{a(-\ln(\mathrm{C}/C_{in}))^{\beta}\,\mathrm{C}} = \int_0^t dt\,. \qquad \text{(A4)}$$

The integral to the left has a simple analytical solution for $\beta \neq 1$:

$$-\frac{1}{a}\int \frac{(-\ln(\mathrm{C}/C_{in})^{-\beta}}{\mathrm{C}}\,d\,\mathrm{C} = \frac{(-\ln(\mathrm{C}/C_{in}))^{1-\beta}}{a(1-\beta)} \qquad \text{(A5)}$$

Using (A5) in Eq. (A4) and noting that the lower integration limits on both sides vanish, the following is obtained:

$$(-\ln(\mathrm{C}/C_{in}))^{1-\beta} = a(1-\beta)t\,. \qquad \text{(A6)}$$

Raising to the power 1/(1-β) followed by exponentiation, we get the final expression:

$$\mathrm{C} = C_{in}\exp\!\left[-(a(1-\beta)t)^{1/(1-\beta)}\right]. \qquad \text{(A7)}$$

With $a > 0$ and $\beta < 1$, and expressing it in terms of A/Q in the same way as in Sec. II, this is the Matts-Öhnfeldt equation. Identification gives $k = 1/(1-\beta)$ and $w_k = a(1-\beta)V/A$.